ROUND I: Number Theory - LCM, GCF, primes, and bases

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. What base must this addition problem be in?

$$\begin{array}{r} 142 \\ +\ 221 \\ \hline 403 \end{array}$$

2. What is the difference between the sum of all eight positive integral divisors of 66 and the sum of all eight positive integral divisors of 70?

3. P boxes are arranged in a circle and numbered consecutively one through P, where P is a prime number $\geq$ 5. A ball is put in the first box, the fifth box, and every 4th box thereafter moving around the circle. In terms of P, which box is the last to get a ball?

ANSWERS
(1 pt.) 1._____

(2 pts) 2._____

(3 pts) 3._____

Doherty, Mass. Academy

ROUND II: Algebra I - OPEN

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. Part of a multiplication table is shown.
   For example, $nz = 255$. What is the value of B–A?

	z
n	255
$n+1$	272
$n+2$	A
$n+3$	306
$n+4$	B
$n+5$	340

2. Find the least positive integer for which the sum of its reciprocal and $\dfrac{3}{7}$ is greater than 3 times its reciprocal.

3. Six years less than four times Sue's age is half Jim's age. When their combined ages total 108, Jim will be five times the age Sue will be then. How old are Jim and Sue now?

ANSWERS
(1 pt.) 1._____

(2 pts) 2._____

(3 pts) 3. _Jim_____, _Sue_____
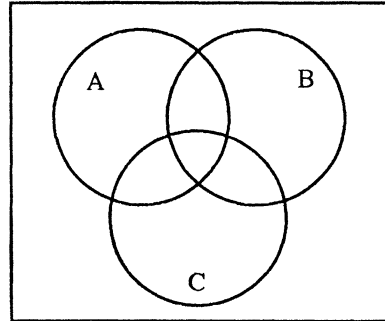
Bromfield, Quaboag, Tahanto

ROUND III: Set Theory

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

NOTE: $\overline{A}$ denotes the comlement of set A.

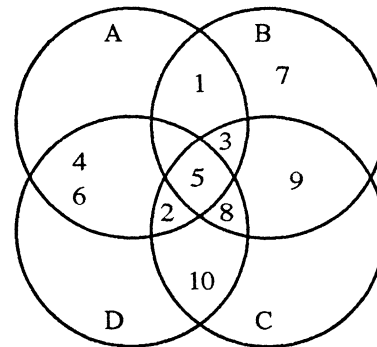1. On the Venn Diagram like this in the
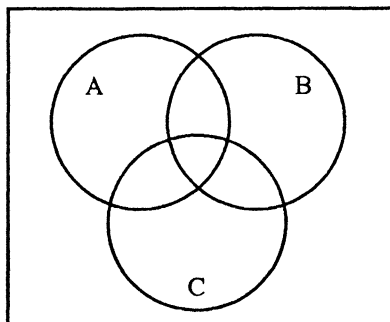   answer space shade
   $$\overline{A} \cap (B \cup C)$$

2. The number of subsets of set S is 28 more than the number of subsets of set T. How many elements are in set S?

3. Let   A = {1,2,3,4,5,6}
         B = {1,3,5,7,9}
         C = {2,3,5,8,9}
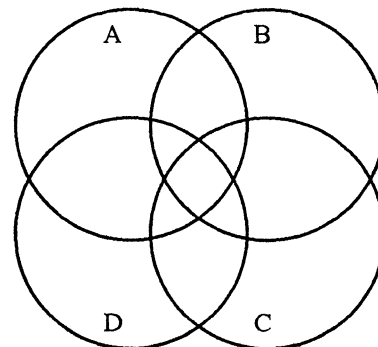   D = (2,4,6,8,10}

   Relocate correctly all the numbers which are
   wrongly located at the right, and only those,
   on the answer section diagram.
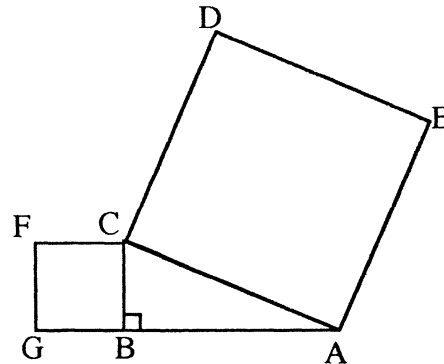
ANSWERS
(1 pt.) 1.

(3 pts) 3.

(2 pts) 2._____

Hudson, Notre Dame, Shrewsbury

ROUND IV: Measurement

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM OR AS DECIMALS ROUNDED TO 3 PLACES TO
THE RIGHT OF THE DECIMAL POINT

1. A circle and a sphere have the same radius. The area of the circle and the volume of the sphere
    have the same numerical value. What is the common radius?

2. If the area of square FCBG is 36 and
    the area of square CDEA is 100,
    find the perimeter of right $\triangle$CBA.

3. Cube A is contained within sphere B, which is contained within Cube C, which is contained
    within sphere D, which is contained within cube E, which is contained within sphere F. If the
    volume of cube A is 8cm^3, the radius of sphere F must be no smaller than what number of cm?

ANSWERS
(1 pt.) 1._____

(2 pts) 2._____

(3 pts) 3._____

Auburn, Clinton, St. John's

ROUND V: Polynomial Equations, Complex Numbers

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM OR AS DECIMALS ROUNDED TO 3 PLACES TO THE RIGHT OF THE DECIMAL POINT

1. Solve $2x - \left(\dfrac{4}{5}\right)^{(i^{434})} = 0$, where $i = \sqrt{-1}$.

2. The conjugate of a complex number $a + bi$ is $a - bi$ and is denoted by $\overline{a + bi}$.

Evaluate $(\bar{3} + \bar{i})^2 (\overline{3 - i})^3$, putting your answer into $a + bi$ form.

3. If p and q are the roots of $x^2 - 5x + 2 = 0$,
Evaluate
$$p^2 + 4pq + 2q^2 - 5q + 2$$

ANSWERS
(1 pt.) 1._____

(2 pts) 2._____

(3 pts) 3._____

Auburn, St. John's, South

TEAM ROUND: Topics of previous rounds and open

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM AND ON THE SEPARATE TEAM ROUND
ANSWER SHEET.   2 points each.

1. What is the smallest possible sum of two distinct positive numbers each of which is divisible
    by both 52 and 90?

2. Suppose $f(x) = \dfrac{x}{1-x}$. If $f(f(2)) = f^2(2)$, $f(f(f(2))) = f^3(2)$, etc.,
    Find the numerical fraction represented by $f^{100}(\#)$.
    2

3. In a group of 120 students, 60 are enrolled in Spanish and 56 in French, but 8 are taking both
   Spanish and French. What percent of the total group is taking neither Spanish nor French?

4. Determine the largest number of whole bricks of dimension 2 x 2 x 3
    that can be fit inside a box 3 x 4 x 5.

5. Two roots of the equation $x^3 + ax^2 - 12x + c = 0$ are 2 and 6. Find the third root.

6. Find the greatest possible value of the determinant for the matrix
$$\begin{bmatrix} 7 & x+1 \\ x+4 & x+2 \end{bmatrix}$$

7. The expression $\dfrac{3x-5}{x+7}$ can be expressed in the form $A + \dfrac{B}{x+7}$, where A and B are integers.
    Find the value of A + B.
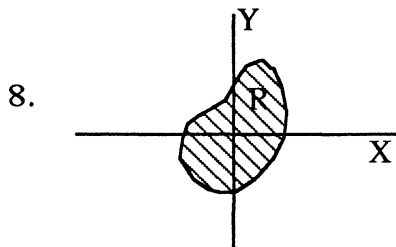
8.
    Region R has area 15. What is the area of region T which
    consists of all points ($2x - 2$, $2y + 5$), where (x,y) is in R?
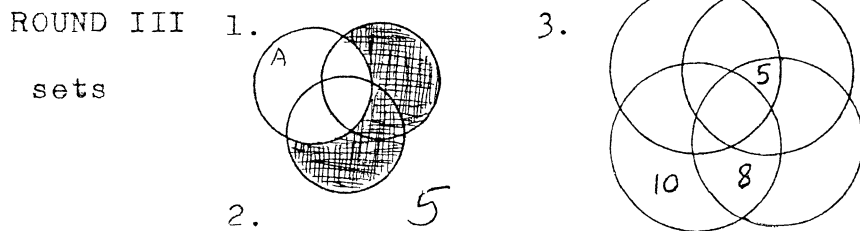
9. Solve for x:
$$5^{2x+2} + 1 = 5^{x+2} + 5^x$$

Auburn, Bancroft, Hudson, Mass. Academy, Quaboag, St. John's, Worcester Academy

ROUND I   1.   1 pt   6

thry

             2.   2 pts   0 or zero

             3.   3 pts   P-3 rd

ROUND II   1.   1 pt   34

alg 1

             2.   2 pts   5

             3.   3 pts   Jim 84, Sue 12

ROUND III   1.   3.

sets

             2.   5

ROUND IV   1. 1 pt $\frac{3}{4}$ or .75

meas

             2. 2 pts   24

             3. 3 pts   ~~6√3~~ or ~~10.392~~
                          $3\sqrt{3}$ 5.196

ROUND V   1. 1 pt $\frac{5}{8}$ or .625

poly eq

             2. 2 pts   300 + 100 i

             3. 3 pts   29

TEAM ROUND
2 pts each

1.   7020

2. $-\dfrac{2}{199}$

3.   10 %

4.   4

5.   −3

6.   11

7.   −13

8.   60

9.   −2, 0   need both